

Adjusting the Putt Distance on Inclined Surfaces with respect to a Flat Putt

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Objective

- There are many different heuristic approaches for distance control in putting (e.g. *Dave Pelz's Putting Bible*).
- However, it seems that there is still a need for a unified and scientifically based concept, that can be used in practice.
- As the ball passes through different phases before the onset of pure roll, depending on various parameters like green speed, inclination and angles and many individual characteristics, we propose a correction with respect to a putt on a plane surface.
- Furthermore, we estimate the effect of the sliding phase on the proposed correction formulas.
- With *Quintic Ball Roll* and *Overhead Putt Tracker*, the validity of the correction formulas can be tested experimentally.

Methods

- The correction method is based on the physics of a rolling ball in the presence of sliding friction, rolling friction and inclination.
- Given these parameters, the total length of the putt depends obviously on the initial velocity (v_0) of the ball.
- ... but also on the distance the ball flies after impact, which depends on individual features such as the loft of the club-head, the execution of the stroke etc.

→ **Concept of corrected length s_{corr} .**

“How long would a putt be on a plane surface, if it were hit with the same initial velocity that is needed to reach a length s , given the actual inclination and green speed.”

Methods ctd.

For this talk, we restrict ourselves to a one-dimensional motion (along the fall line) in the presence of sliding friction (μ_S), rolling friction (μ_R) and inclination (θ).

- The coefficient of rolling friction (μ_R) can be derived from the distance measured by a *Stimp-meter* (D_{Stimp}):
- $$\mu_R = \frac{7v_{0;S}^2}{10gD_{Stimp}}.$$
- $v_{0;S}$: Initial velocity from Stimp-meter ($1.83ms^{-1}$).
- g : Acceleration of gravity ($9.81ms^{-2}$).

Dynamic of a Golf Ball on an Inclined Surface

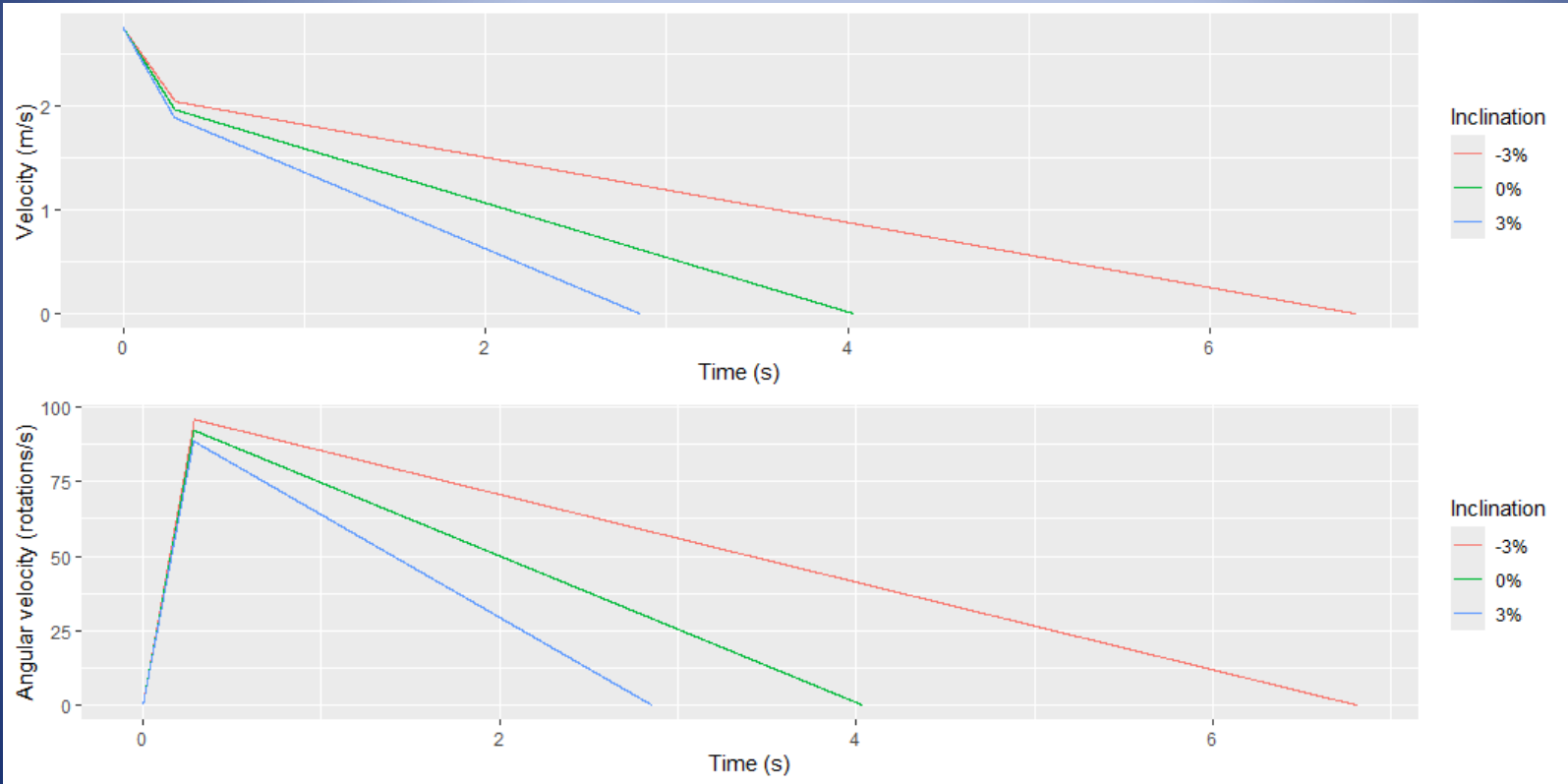
- The ball slides until the condition of pure rolling has been reached: $v = \omega \times r_b$.
- ω : angular velocity, r_b : radius of ball.
- Assumption: (Initial angular velocity $\omega_0 = \omega(t = 0) = 0$)
- → equations of motion:

$$\ddot{\phi} = \dot{\omega} = \begin{cases} \frac{g}{r_b} \left(\frac{5\mu_G}{2} - \frac{5\theta}{7} \right) & \text{for } t \leq t_R \quad (v \geq \omega r_b) \\ \dot{v}/r_b & \text{for } t > t_R \end{cases}$$
$$\ddot{x} = \dot{v} = \begin{cases} -g(\mu_G + \theta) & \text{for } t \leq t_R \quad (v \geq \omega r_b) \\ -\frac{5g}{7}(\mu_R + \theta) & \text{for } t > t_R \end{cases}$$

- t_R : Time when pure rolling begins: $\frac{v_0}{g} \left(\frac{14}{49\mu_G + 4\theta} \right)$

Velocity and Angular Velocity of the Ball vs. Time in Seconds According to Model

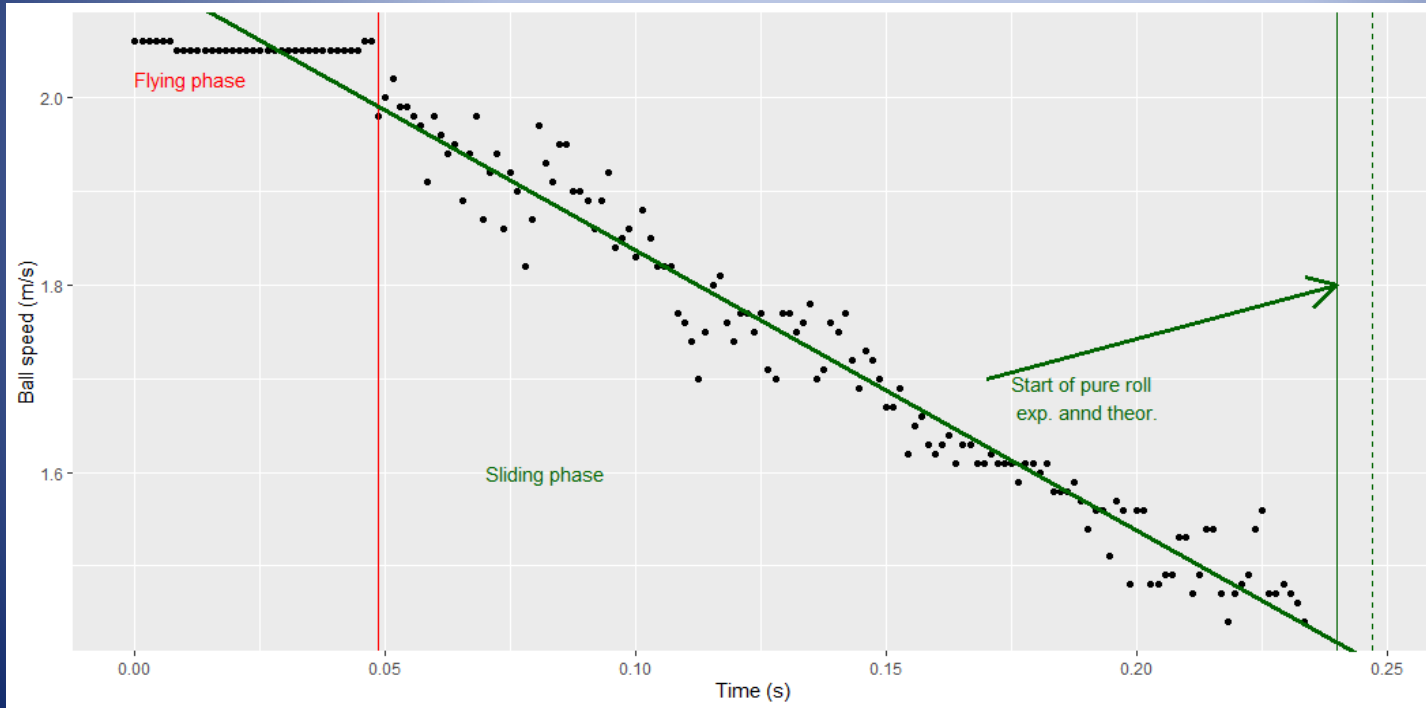
Stimp-distance: 10.5 ft, $\mu_G = 0.28$, $v_0 = 2.75$ m/s



Comparing with Experimental Measurements

Data from a Single Putt

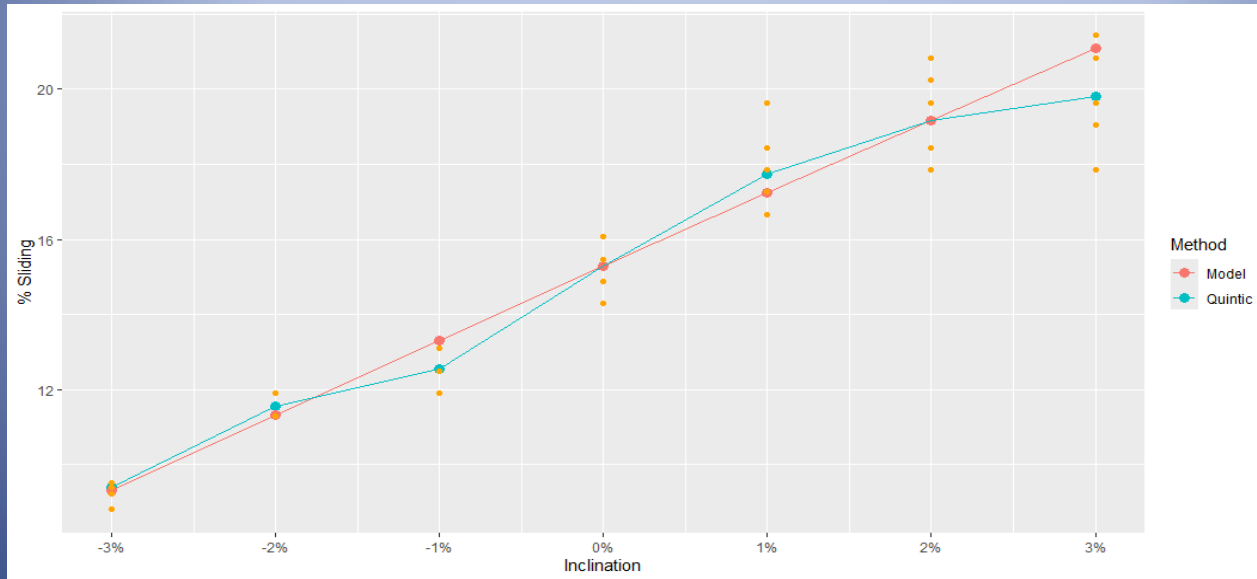
- Data of putt captured by Quintic Ball Roll.
- Putt: 3% downhill, $D_{Stimp} = 10.5$ ft.



- y-axis: ball speed after impact
- x-axis: time in seconds

Percentage of Sliding Phase - Measured Values and Model Predictions

- Mean values taken from 10 putts on an artificial surface with speed of 10.5 ft (Stimp meter distance) at various inclinations.
- -3% inclination: 8% Sliding 92% Rolling
- +3% inclination: 20% Sliding 80% Rolling
- Ratio of Sliding and Rolling phases is influenced heavily by surface inclination!



Length Correction

- By splitting the whole putting trajectory into the sliding phase and the rolling phase, it is straightforward to calculate a correction factor.
- However, that correction depends on the coefficient of sliding friction μ_S or the ratio λ between gliding friction and rolling friction
- $\lambda = \mu_S / \mu_R$, which is commonly not measured at golf courses.

→ How accurate is the correction, when considering only rolling motion?

Derivation of the Length Correction Considering Sliding and Rolling Phase

- θ : Inclination. (θ = tangent of inclination angle in radians)
- v_0 : Initial velocity of the ball after impact with club head.
- t_R : Time of onset of pure rolling. $\frac{v_0}{g} \frac{14}{49\mu_S + 4\theta}$
- v_1 : Velocity of the ball at onset of pure rolling. $v_0 - \left(1 - \frac{14(\mu_S + \theta)}{49\mu_S + 4\theta}\right) v_0$
- S_S : Distance covered in sliding phase. $v_0 t_R - g(\theta + \mu_S) \frac{t_R^2}{2}$
- S_R : Distance covered in rolling phase. $\frac{7}{10} \frac{v_1^2}{g(\mu_R + \theta)}$

$$\rightarrow S_{tot} = S_S + S_R$$

- $S_0 = S_0(v_0, \theta, \mu_S, \mu_R)$: Total distance on plane surface ($\theta = 0$).

$$S_0 = \frac{v_0^2}{g} \left(\frac{24\mu_G + 35\mu_S}{98\mu_G\mu_R} \right)$$

$$\rightarrow \text{Correction factor: } c_{SR} = S_{tot}/S_0$$

Length Correction Considering Sliding and Rolling (Phase II)

S_{tot} : Total distance (sliding phase + rolling phase).

$$S_{tot} = \frac{7v_0^2(8\theta^2 + 4\theta(7\mu_S - 3\mu_R) + 7\mu_S(35\mu_S + 24\mu_R))}{2g(4\theta + 49\mu_S)^2(\theta + \mu_R)}$$

→ Correction factor: $c_{SR} = S_{tot}/S_0$

$$1/c_{SR} = \frac{343\mu_S\mu_R(8\theta^2 + 4\theta(7\mu_S - 3\mu_R) + 7\mu_S(35\mu_S + 24\mu_R))}{(4\theta + 49\mu_S)^2(\theta + \mu_R)(35\mu_S + 24\mu_R)}$$

Length Correction Considering only the Rolling Phase

- When considering the whole trajectory as “pure roll” the correction formula is very simple:
- The initial speed in order to achieve a total length s considering only rolling motion is given by:

$$v_0(s, \theta) = \sqrt{\frac{10}{7} s g (\mu_R + \theta)}$$

- By setting:
- $v_0(s, \theta) = v_0(s_{corr}, \theta = 0)$
- It follows:

$$s_{corr} = c_R \times s \quad \text{and} \quad c_R = 1 + \frac{\theta}{\mu_R} \quad \theta \text{ in radians}$$

Error in the Correction Factor when Considering only Pure Roll

- c_{SR} correction factor considering both, sliding and rolling phase.
- Table: Maximum difference between c_{SR} and c_R (in %).

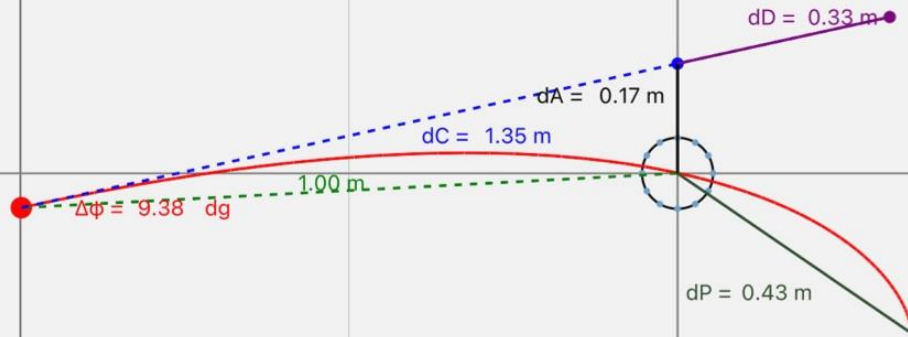
D_s (ft)	lambda	maximum error
8	3	1.19%
8	4	0.81%
8	5	0.60%
8	6	0.48%
10	3	1.50%
10	4	1.02%
10	5	0.76%
10	6	0.60%
12	3	1.82%
12	4	1.23%
12	5	0.92%
12	6	0.72%

Conclusions

- Concept of length correction *with respect to a putt on a plane surface* is useful and easy to apply.
- Model in excellent agreement with experimental measurements.
- Length corrections derived only considering the rolling phase are more than sufficient for practical purposes.

Further Developments

- Equations of motion where there is an angle between the fall line and the line ball-hole.
- → Curved trajectories.
- The corrected distance and curved trajectories are now implemented in **www.puttalyze-app.com**



References

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